KTH Teknikvetenskap

SF2729 GROUPS AND RINGS
HOMEWORK ASSIGNMENT I
GROUPS

The following homework problems can count as the first problem in the first section of the mid term exam and in the final exam. The solutions should be handed in no later than on February 16. The computations and arguments should be easy to follow. Collaborations should be clearly stated.

Credits on homework	31-35	26-30	21-25	16-20	11-15	6-10	0-5
Credits on problem 1 of part I	6	5	4	3	2	1	0

Problem 1. Show that $[A, B] = AB - BA$ defines a binary operation on the set of real skew-symmetric $n \times n$-matrices. Futhermore, show that for $n = 3$, this binary structure is isomorphic to the binary structure given by the vector product on $\mathbb{R}^3$. **(5)**

Problem 2. Verify that the two sets of matrices $\{A_0, A_1, \ldots, A_{n-1}, B_0, B_1, \ldots, B_{n-1}\} \subseteq \mathrm{GL}_2(\mathbb{R})$ and $\{C_0, C_1, \ldots, C_{n-1}, D_0, D_1, \ldots, D_{n-1}\} \subseteq \mathrm{GL}_2(\mathbb{C})$ form isomorphic groups, where

$$A_j = \begin{pmatrix} \cos j\phi & -\sin j\phi \\ \sin j\phi & \cos j\phi \end{pmatrix} \quad \text{and} \quad B_j = \begin{pmatrix} \sin j\phi & \cos j\phi \\ \cos j\phi & -\sin j\phi \end{pmatrix}$$

$$C_j = \begin{pmatrix} \xi^j & 0 \\ 0 & \xi^{-j} \end{pmatrix} \quad \text{and} \quad D_j = \begin{pmatrix} 0 & \xi^{-j} \\ \xi^j & 0 \end{pmatrix},$$

for $j = 0, 1, \ldots, n-1$, where $\phi = 2\pi/n$ and $\xi = e^{i\phi}$ is a primitive root of unity. **(5)**

Problem 3. Show that every subgroup of S_5 of order 6 is isomorphic to S_3. [1] **(5)**

Problem 4. Determine the order of the subgroup of S_5 generated by (123) and (345). **(5)**

Problem 5. Show that an associative binary structure on a set S which has a left unit and a left inverse of any element a is in fact a group, i.e., has a two-sided unit and a two-sided inverse of any element. **(5)**

Problem 6. Let $G = \mathrm{Gl}_2(\mathbb{F}_2)$ be the general linear group over the field $\mathbb{F}_2$ with two elements. [2] Choose a nice generator set for G and use it to draw the Cayley digraph for G. **(5)**

[1] Not true for S_5 but for S_4 and A_5. Choose any of those to replace S_5.

[2] $\mathbb{F}_2$ can be thought of as $\mathbb{Z}_2$, i.e., the integers modulo 2.

Problem 7. Show that the set H of upper triangular matrices form a subgroup in the general linear group $\mathrm{Gl}_n(\mathbb{R})$ of invertible real $n \times n$-matrices. Furthermore, show that H has no non-trivial elements of finite order. [3] **(5)**

[3]This is not possible to prove, since there are elements of order 2. Howerver, you can show that there are no elements of higher order.